The Ensemble Fractal Dimension of Clouds: Total Perimeter, Area, and Resolution

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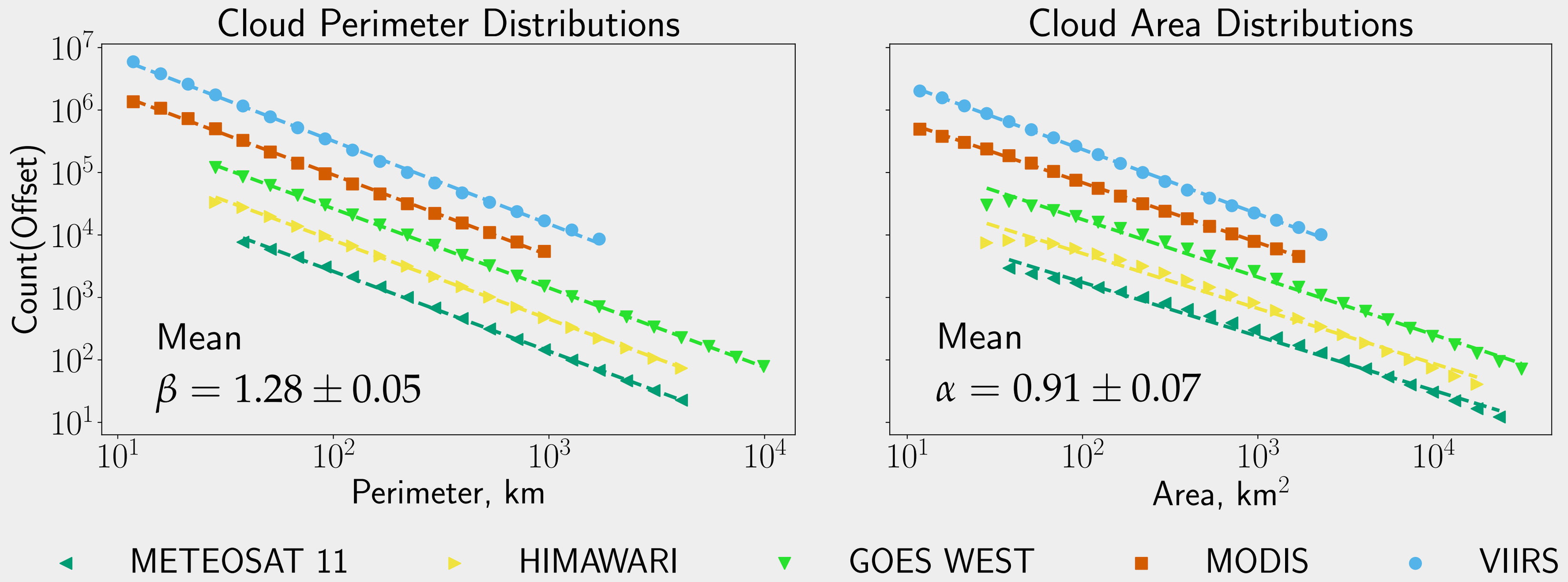


Figure 1: Cloud perimeter distributions (left; equation 1) and area distributions (right; equation 2) measured from satellite data. Distributions are vertically offset for clarity. Dashed lines are linear regressions. The slope is the negative value of β (left; equation 1) or α (right; equation 2).

We derive relationships between total cloud perimeter, measurement resolution, and total cloud area.

Cloud perimeters p and areas a are well described by power law distributions n_p and n_a (figure 1)

$$n_p \propto p^{-\beta-1}, \qquad \beta > 1. \tag{1}$$

$$n_a \propto a^{-\alpha-1}, \qquad 0 < \alpha < 1, \tag{2}$$

The change in total cloud perimeter with pixel size ξ is broken up into two orthogonal components

1. Perimeter lengths changing due to the individual fractal dimension $p \propto \xi^{1-D_i}$ with $D_i \approx 4/3$ (See the top row in figure 2).
2. The smallest measurable perimeter changes (Note the small clouds in the bottom row of figure 2).

Accounting for these effects, expressions relating the total perimeter P and total number of clouds N to pixel size ξ can be derived (see additional details)

$$N \propto \xi^{-\beta D_i}, \tag{3}$$

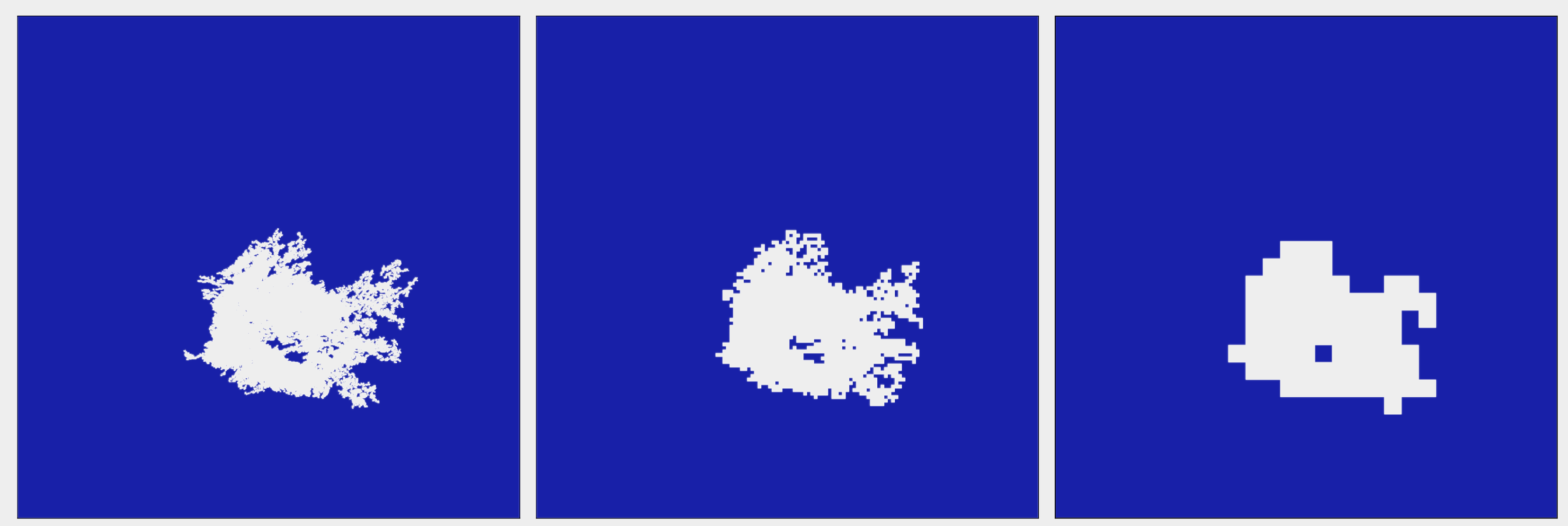
$$P \propto \xi^{1-\beta D_i}. \tag{4}$$

Equation 4 implies an "ensemble" fractal dimension $D_e = \beta D_i \approx 1.70$, applicable to total cloud perimeter, larger than the traditional fractal dimension D_i, applicable to individual cloud perimeters.

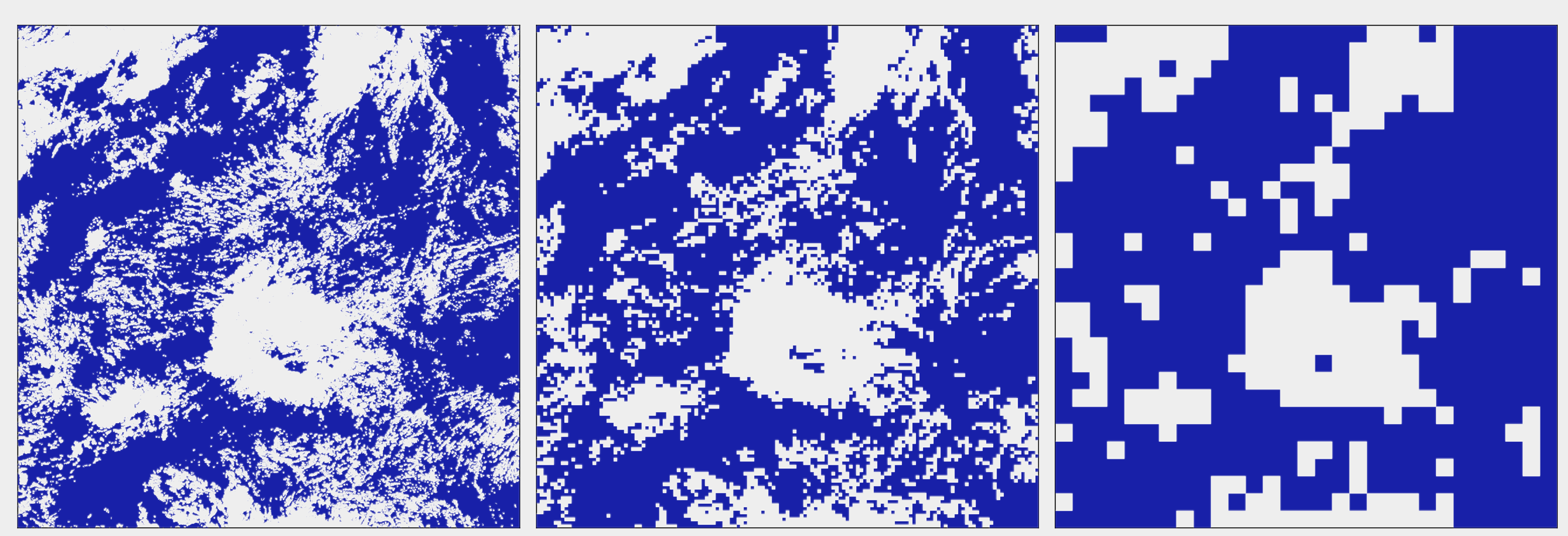
The fractal dimension of the cloud ensemble is larger than the fractal dimension of a single cloud.

Total cloud perimeter is expected to increase in a warmer climate. How will total cloud area respond?

- A recent study derived an expression for total cloud perimeter as a function of atmospheric stability (Garrett et al., 2018) and predicted total cloud perimeter will increase with a warmer more stable climate.
- Individual cloud perimeters and areas are related by the fractal dimension D_i, but it is currently unknown how total cloud perimeter relates to total cloud area.
- We find total cloud perimeter and total cloud area depends on the smallest and largest clouds. Can global climate models (GCMs) produce the correct statistics without resolving the smallest clouds, which are potentially of millimeter scales?



A single cloud perimeter scales with resolution.



The total number of clouds in the ensemble scales with resolution.

Figure 2: The cloud mask product from GOES-WEST under various measurement resolutions. Artificial resolution coarsening is achieved by grouping pixel clusters and averaging the grouped pixels.

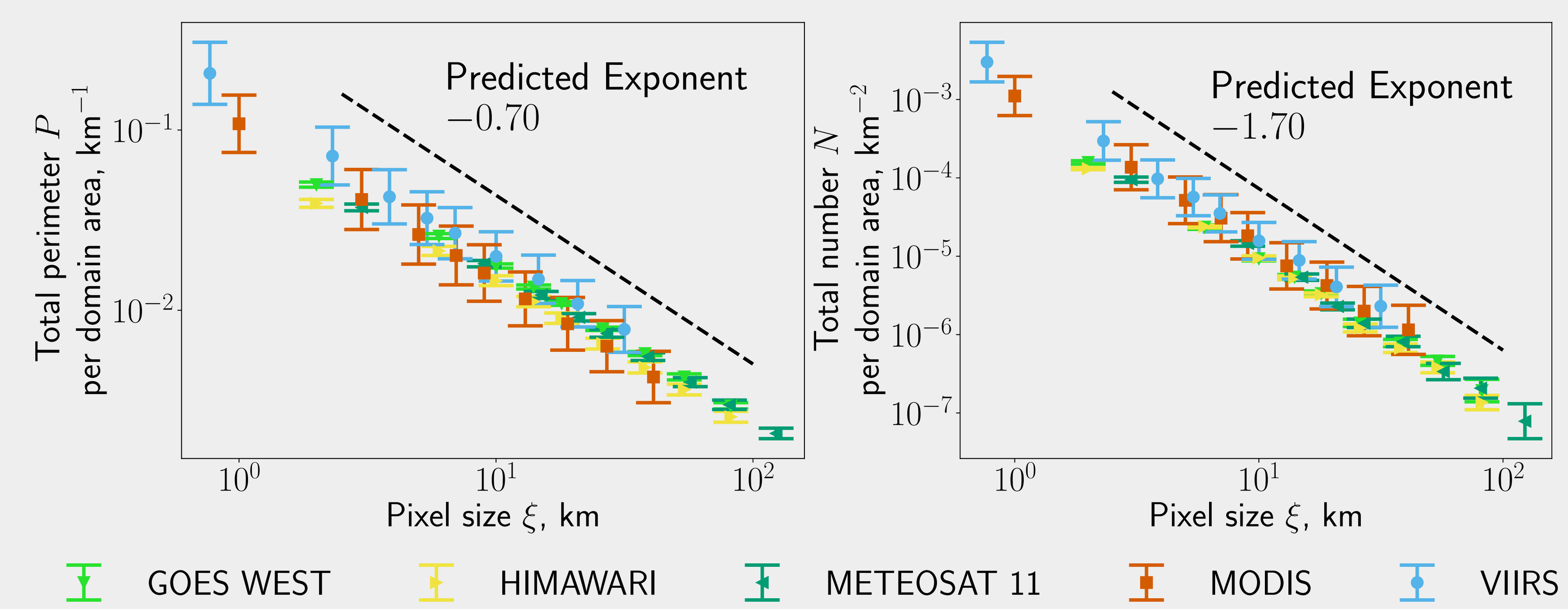


Figure 3: Left: total cloud perimeter P, normalized by domain area, as a function of resolution. Right: total number of clouds N, normalized by domain area, as a function of resolution. Different resolutions were achieved by artificially coarsening the satellite resolution as in figure 2. "Predicted Exponent" uses equations 4 and 3, respectively, for the left and right plots using $D_i = 4/3$ and $\beta = 1.28$.

Predictions of how total perimeter scales with resolution agree with satellite data.

Total cloud area A and total cloud perimeter P are proportional:

$$A = P\frac{\alpha(\beta-1)\, a_{\mathrm{min}}^{\alpha}}{\beta(1-\alpha)\, p_{\mathrm{min}}} a_{\mathrm{max}}^{-\alpha+1}, \tag{5}$$

or simply

$$A \propto P. \tag{6}$$

If total cloud perimeter increases, as predicted, total cloud area will increase proportionally.

Key Assumptions

- Cloud perimeters and areas follow a power law over a finite but large range of scales (as shown in figure 1).
- Clouds with perimeters or areas outside this range of scales contribute negligible perimeter/area to the total.
- Clouds within the power law distribution have, on average, a constant fractal dimension D_i. This is implied by the first assumption.
- The effects of clouds combining or separating as the pixel size ξ changes are negligible or implicit in measurements of the individual fractal dimension D_i.

References

Garrett et al., 2018:
Thermodynamic constraints on the size distributions of tropical clouds.

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Additional Details

https://thomasdewitt.chpc.utah.edu/agu-2022/poster-info.html